Metric Geometry and Gerrymandering

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Many slides from M. Duchin & M. Bernstein
Metric Geometry & Gerrymandering Group

Small team studying applications of math and computing to redistricting
  • Research & publication
  • Interdisciplinary collaboration
  • Outreach & education

sites.tufts.edu/gerrymandr/
Congressional Representation

Census counts people

States divide into districts

Congressional reps are apportioned to states
Redistricting as a Math Problem

Red has 52.5% of the population by 75% of the districts

... tens of thousands of census blocks per district!

Partitioning with attributes

- Black has 45% of the population but 25% of the districts
- Red has 52.5% of the population by 75% of the districts
Goals and Constraints

- **Goals**
  - **Proportionality:** Districts are representative
  - **Gerrymandering:** Partition to extremize an attribute

- **Constraints**
  - Equal population
  - No holes
  - Not too “eccentrically shaped”
Other Values or Principles

- Proportionality
- Competitiveness
- Governability
- Partisan Fairness

...?

Relatively few legal parameters
How to Gerrymander

Intuition:
Any agenda will cause eccentric shapes.

Image from:
A Formula Goes to Court: Partisan Gerrymandering and the Efficiency Gap
Bernstein & Duchin, Notices of the AMS (to appear)
Compactness

COMPACTNESS

- Many metrics exist:
  - Isoperimetry
    \[0 \leq 400\pi A/P^2 \leq 100\]
  - Convexity
    How indented?
  - Dispersion
    How spread out?

but perimeter is problematic
there are many legitimate reasons for non-convexity
and all of this is 19th century mathematics!
“If you drove down the interstate with both car doors open, you’d kill most of the people in the district.”

- Unnamed state legislator

Reported in: “Thomas right to oppose racial ‘homelands’”
(The Item, August 17, 1994)
Q:
What can we do as programmers and computer scientists?
Partners in Redistricting

$10^9$ computations/second
No legal understanding
No sympathy

?? computations/second
Strong legal understanding
Potentially sympathetic
Spectrum

Clearly easy:
• Visualizing districting plans
• Data collection

Clearly difficult:
• Extracting optimal plans

Huge gray area:
• Improving plans
• Evaluating compactness
• Sampling possible plans
Any software extracting the “best possible” districting plan* also resolves the most famous open problem** in computer science.

\[ P \overset{?}{=} NP \]

* under any reasonable metric.
** (News!) Perhaps not open any more.
What Can We Do?

Analysis and comparison
What Can We Do?

Local optimization
Which Objective Function?

- Isoperimetric ratio?
- Graph curvature?
- Dispersion?
- Equal population?
- Minority representation?
- Efficiency gap?

“Pareto optimality”
What Can We Do?

Screenshot from “Quantifying Gerrymandering” (Duke Data+)
https://services.math.duke.edu/projects/gerrymandering/

Sampling/MCMC
Call to action:

We need your help.
About the August Workshop

A Geometry of Redistricting workshop will be offered at Tufts University from August 7-11, 2017, mixing math, law, and civil rights. The first three days of the week (M-W) will be open to the public and made available online. The last two days (Th-F) will be devoted to specialized training, broken down into three tracks for which participants were selected by an application process in early Spring.

Registration for the Monday to Wednesday workshop has now closed. We will accommodate walk-ins after pre-registered participants have been seated.

Schedule for Monday-Wednesday

Here is the full program for Monday-Wednesday, and here is the schedule on its own.
DistrictGenius

https://github.com/gerrymandr/district-genius
A. M’ndange-Pfupfu & V. Archambault:

QGIS Compactness Plugin

https://github.com/gerrymandr/qgis-compactness
mander

Python package for calculating metrics related to district shapes.

Installation

Requires GDAL (brew install gdal) on Mac OS X with Homebrew.
Then:

```
pip install mander
```

Usage

```python
from mander.districts import District
from mander.metrics import calculatePolshyPopper

# Load a district from a GeoJSON or SHP file path
district = District(path='C:/path/to/GeoJSON.json')

# Call a metrics function on the district class object
calculatePolshyPopper(district)
```

Assumptions

The package will accept any projection, but all districts are converted to US National Atlas Equal Area. Equal area projections are best for calculating compactness metrics. This ensures that when calculating area and perimeter measures, each district has a minimal amount of distortion, so districts in different parts of the country can be accurately compared to one another. Therefore, this shouldn't be used for districts outside of the U.S. Future versions will accept local projections.

Compactness Metrics

```python
metrics.calculatePolshyPopper
```

The Polshy-Popper measure is a ratio of the area of the district to the area of a circle whose circumference is equal to the perimeter of the district.

The formula for calculating the Polshy-Popper score is:

\[ 4\pi \times \frac{A}{P^2} \]

Where \( A \) is the area of the district and \( P \) is the perimeter of the district.

https://github.com/gerrymandr/python-mander
Metric Visualizer

https://github.com/gerrymandr/metric_visualizer
Future Workshops

- **Wisconsin**
  October 12-15, 2017

- **North Carolina**
  November 2-5, 2017

- **Texas**
  February 1-4, 2018

- **California**
  March 15-18, 2018

https://sites.tufts.edu/gerrymandr/project/
Potential Projects

- Unglamorous but necessary **data scraping**
- **Gerrymandr**, the app
- Demo and comparison of district **sampling** algorithms
- Illustrate **evolution** of districting plans
- Redistricting **competitions**
- GIS Team **request page**
- **Crowd-sourced** redistricting
- Many more!
Open Questions

■ What is the role of machine learning in redistricting?

■ How complicated is the energy landscape of political redistricting specifically?

■ How do we ensure transparency for redistricting software?
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